

Thermomagnetic instability of standing flux-antiflux front in layered type-II superconductors.

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Abstract

Stability of standing flux-antiflux front in anisotropic layered superconductors is considered. We describe two assisting mechanisms destabilizing the standing vortex-antivortex front. There are anisotropy of the layered superconductors and the heat, released by the vortex dynamics. We present the conditions of the front stability for various anisotropy and heating parameters. We predict that even small vortex-antivortex heating can result in front instability. The characteristic size of the unstable pattern is estimated.

I. INTRODUCTION

Studies of patterns in the magnetic flux distribution in the type-II superconductors are attracting the attention of many research groups [1–4] whose magneto-optical experiments demonstrate that nonuniform flux penetration occurs. Patterns with branch-like structures have been found in most of high T_c materials, like $YBa_2Cu_3O_{7-x}$ [5] and $Bi_2Sr_2CaCu_2O_{8+x}$ [6]. The nucleation of dendrite like patterns in MgB_2 films is another example [7, 8]. These complex structures consist of alternating low and high vortex density regions and are found in a certain temperature window. Likewise, flux penetration in the form of droplets separating areas of different densities of vortices has been observed in $NbSe_2$ [9]. Usually the occurrence of flux patterns in interfacial growth phenomena can be attributed to a diffusion driven, long-wavelength instability of a straight front, similar to the Mullens-Sekerka instability [10] found in crystal growth. The nucleation of nonuniform patterns associated with the propagation of a flux front into a flux-free sample has been attributed to such an interfacial instability. This results from a thermomagnetic coupling [7, 8, 11–13] where a higher temperature leads to a higher vortex mobility, enhanced flux flow, and hence a larger heat generation.

On the other hand, the situation when the vortices interact in a superconductor with the flux of the opposite sign is less theoretically studied. This flux configurations arises for example when a DC bias current creates vortices and antivortices on the opposite side of the superconducting strip [14]. Another example which is now under intensive investigation arises upon exposing the previously magnetized sample to the magnetic field of an opposite direction. According to the experimental data [2–4, 15, 16], the boundary between vortices and antivortices exhibits many features suggestive of a long wavelength instability. The cause of the instability at the boundary between fluxes of opposite sign is still being debated. In particular, Fisher et al. [17, 18] proposed a non thermomagnetic mechanism of instability caused by an in-plane anisotropy of the vortex mobility. This mechanism of instability was carefully reinvestigate by the van Saarloos et al [19]. They confirmed the finding of Fisher et al. [17, 18] that standing vortex-antivortex fronts have an instability to a modulated state, while the moving fronts were found to be stable for all anisotropies. In fact however, the flux-antiflux instability was experimentally detected in a system with small [4] and moderate anisotropy [3]. Several years latter this model was improved by an additional assumption of a step shape and anisotropy of the voltage current characteristics [18] and explained the

experimental result in moderate anisotropic superconductor $YBa_2Cu_3O_{7-\delta}$. Unfortunately this assumption cannot explain the instability in pure isotropic systems like Nb and MgB_2 . On the other hand the thermomagnetic mechanism can also be responsible for flux-antiflux instability when the flux-antiflux front is heated both by the vortex (antivortex) dynamics and by the vortex-antivortex annihilation.

In this paper we report on the thermomagnetic theory of the flux-antiflux instability in anisotropic layered superconductor.

II. MODEL AND BASIC EQUATIONS

We start with a model of two-component vortex gas [20] spatially homogeneous along the z axes, which is valid for the experimentally interesting situation of the low magnetic field when typical spacing between vortices a_0 essentially exceeds vortex-vortex (antivortex) interaction radius ξ , and the vortex velocity depends only on the edge screening current that is assumed to be homogeneously distributed across the sample. The vortex-vortex repulsion, in this case, keeping the number of vortices, cannot play a significant role. One must take into account both vortex-antivortex annihilation and heat release accompanying this process. We should also take into consideration heat absorption by the sample lattice in order to prevent the rise of unlimited temperature.

The vortex-antivortex annihilation obeys the well-known master equations of the recombination theory [21, 22]

$$\frac{\partial n_+}{\partial t} + \nabla (n_+ \mathbf{v}_+) + g n_+ n_- = 0, \quad (1)$$

$$\frac{\partial n_-}{\partial t} + \nabla (n_- \mathbf{v}_-) + g n_+ n_- = 0 \quad (2)$$

$$g = \xi v, \quad v = \text{mod}(\mathbf{v}_+ - \mathbf{v}_-) \quad (3)$$

where n_+ and n_- are the vortex and antivortex densities, respectively, g is the ratio of recombination for vortices and antivortices, ξ is the cross section of the annihilation, which is of the order of the coherence length of the superconductor, and \mathbf{v}_\pm are the opposite directed vortex-antivortex velocities, which in the creep regime are strongly temperature dependent:

$$\text{mod}(\mathbf{v}_\pm) = v_\pm = v_{\pm FF} \exp\left(-\frac{U}{T}\right).$$

$$\mathbf{J} = \frac{c}{4\pi} \nabla \times \mathbf{B}, \mathbf{B} = \varphi_0 (n_+ - n_-), \mathbf{v}_{\pm FF} = \frac{\mathbf{J} \times \varphi_0}{\eta c}. \quad (4)$$

Here, U is a temperature-dependent pinning potential, φ_0 is the unit flux, \mathbf{J} is the electric current, and η is the viscosity of the vortices, which in an anisotropic system is a tensor with different in plane and across the plane tensor components $\eta_{\alpha\beta}$.

(It should be noted that our master (recombination) equations account numbers of the *topological charges* (vortex/antivortex cores). The number of the vortices and antivortices is changed when they meet each other at the distance of the order of the coherence length ξ , rather than the penetration length of the magnetic field λ . For example, the same equations describe vortex-antivortex annihilation both in the superfluid He4 and in superconducting films where the magnetic field is uniform).

We wish to investigate an anisotropic system where the vortices velocity is not necessarily parallel or perpendicular to the layers of the superconductor. Therefore, we shall take the vortices velocity in a general form

$$v_\alpha = -\gamma_{\alpha\beta} \left(\frac{\partial B}{\partial x_\beta} \right). \quad (5)$$

$$\alpha, \beta = x, y$$

Here $\gamma = \varphi_0/4\pi\eta$ when

$$\gamma = \eta^{-1} = \eta_0^{-1} \begin{pmatrix} \cos^2 \vartheta + \alpha \sin^2 \vartheta & \cos \vartheta \sin \vartheta (1 - \alpha) \\ \cos \vartheta \sin \vartheta (1 - \alpha) & \alpha \cos^2 \vartheta + \sin^2 \vartheta \end{pmatrix} \quad (6)$$

is the inverse tensor of the vortex viscosity [19], α is the anisotropy parameter of the system ($0 < \alpha < 1$), ϑ is the angle between the x axis and the $a - b$ plain of the layered structure (Fig. 1).

Assuming that the heat diffusion length exceeds the width of the slab [12] one can complete the set of equations Eqs. (1) and (2) by the temperature transfer equation in the form

$$\frac{\partial T}{\partial t} = \frac{\kappa_T}{C_p} \Delta T + \frac{1}{C_p} \frac{\delta Q}{\delta t} - \frac{(T - T_0)}{t_R}, \quad (7)$$

where

$$\frac{\partial Q}{\partial t} = W_J + W_A \quad (8)$$

$$W_J = \frac{\eta v^2}{2} (n_+ + n_-); W_A = \xi v \frac{n_+ n_-}{C_p} Q_0 \quad (9)$$

is determined by the energy released both by vortex-antivortex dynamics W_J and by vortex-antivortex annihilation W_A . Here κ_T is the heat conductivity, C_p is the heat capacity, T_0 is the coolant temperature, Q_0 is heat released by annihilation of a single vortex-antivortex pair per unit vortex length, and t_R is the characteristic time of temperature relaxation.

The set of Eqs. (1)-(7) completed by the boundary conditions describes all features of the model.

III. SPATIAL DISTRIBUTION OF FLUX-ANTIFLUX DENSITIES

We consider the case of a restricted sample of length D . In this case, the flux-antiflux interface in the stationary state is formed due to a balance between flux-antiflux entering the sample from the opposite sides and their annihilation in the middle point.

Introducing new variables

$$n_+/n_m = N_+, \quad n_-/n_m = N_-, \quad x/\Delta L \rightarrow x,$$

$$\Delta L = (n_m \xi)^{-1}, \quad t/t_0 = t^*, \quad t_0 = \frac{\eta (\Delta L)^2}{n_m \varphi_0^2}, \quad (10)$$

$$b = N_+ - N_-, \quad N = N_+ + N_-, \quad (11)$$

where n_m is flux density at the interface point, and ΔL is the characteristic width of the region in which the spatial distributions of vortex and antivortex flux densities overlap, forming the interlayer where the vortices of the opposite signs coexist. ΔL may be estimated as $\Delta L \simeq v/gn_m \sim (n_m \xi)^{-1}$ (see Ref. [23]), which is a microscopically large area where the total magnetic induction is suppressed.

One obtain from Eqs. (1)-(2)

$$\frac{\partial b}{\partial t^*} = \frac{\partial}{\partial x} \left(N \frac{\partial b}{\partial x} \right), \quad (12)$$

$$\frac{\partial N}{\partial t^*} = \frac{\partial}{\partial x} \left(b \frac{\partial b}{\partial x} \right) - |N^2 - b^2| \left| \frac{\partial b}{\partial x} \right|, \quad (13)$$

where b is the dimensionless magnetic induction.

In the stationary state we get for N_0 and b_0

$$\frac{\partial}{\partial x} \left(N_0 \frac{\partial b_0}{\partial x} \right) = 0 \quad (14)$$

$$\frac{\partial}{\partial x} \left(b_0 \frac{\partial b_0}{\partial x} \right) - (N_0^2 - b_0^2) \left| \frac{\partial b_0}{\partial x} \right| = 0 \quad (15)$$

Performing the integration in Eq. (14) one obtain

$$N_0 \frac{\partial b_0}{\partial x} = -I. \quad (16)$$

here I is a constant.

Substituting N from Eq. (16) into Eq. (15) and performing the integration we immediately obtain the differential equation for b in the form

$$-W + \frac{I}{2} \ln \left| \frac{I+W}{I-W} \right| = -\frac{b_0^3}{3}. \quad (17)$$

where

$$W = b_0 \frac{\partial b_0}{\partial x}. \quad (18)$$

A. Flux-antiflux Interface

This equation may be solved analytically close to the interface line where b_0 goes to zero.

Assuming that the vortices and antivortices appear at the edges of the samples separated by the distance D (in dimensionless units) and assuming the following boundary conditions

$$\begin{aligned} b_0 \frac{\partial b_0}{\partial x} &= -I, \quad N_{0-} = 0 \text{ at } x = -\frac{D}{2}, \\ b_0 \frac{\partial b_0}{\partial x} &= I, \quad N_{0+} = 0 \text{ at } x = \frac{D}{2}, \end{aligned} \quad (19)$$

we obtain an asymptotically exact result for magnetic induction and vortices density at the interface: Looking for the solution in the vicinity of the flux-antiflux front ($X \rightarrow 0$) in the form

$$b_0 \simeq a_1 x + a_2 x^3; \quad N_0 \simeq a_3 + a_4 x^2 \quad (20)$$

one obtains from Eqs. (16)-(18) to the main order

$$a_1 = -I^{2/3}, \quad a_2 = \frac{I^{4/3}}{180}, \quad a_3 = I^{1/3}, \quad a_4 = \frac{I}{60} \quad (21)$$

It should be noted numerical simulation show that these formulas are valid in a much wider region at front ($b_0 = 0$) and can be considered as an interpolation ones.

Assuming that the slope of the magnetic induction at the front $I \ll 1$ is small one obtains for the characteristic size of the vortex-antivortex area x_c , where vortices and antivortices coexist $N_0(x_c) = b_0(x_c)$ (see Eq.(14)) (see Fig.2).

$$x_c = I^{-1/3} \gg 1 \quad (22)$$

The dimensionless vortex velocity at the interface $u_0 = I^{2/3}$ is a constant. Returning to the dimension variables, we obtain for interface flux velocity

$$u_{\pm} \approx \frac{n_m^2 \xi \varphi_0^2 \exp(-U/T)}{4\pi \eta} I^{2/3}, \quad (23)$$

IV. OVERHEATING INSTABILITY

Let us consider the stability of the vortex-antivortex interface with respect to small deviations from its initial plane shape. For this we shall take the Eqs. (12), (13), and (7) in the more general form

$$\frac{\partial b}{\partial t} - \nabla(N\mathbf{v}) = 0 \quad (24)$$

$$\frac{\partial N}{\partial t} - \nabla(b\mathbf{v}) + (N^2 - b^2)|v| = 0 \quad (25)$$

$$\frac{\partial \Theta}{\partial t} - \kappa \nabla^2 \Theta - w_A - w_J + r(\Theta - 1) = 0 \quad (26)$$

where

$$w_A = (N^2 - b^2) S_A |v|, w_J = N S_J v^2 \quad (27)$$

are the dimensionless annihilation and Joule heat terms.

Here $S_A \equiv (Q_0 n_m / 4\pi T_0 C_p)$; $S_J = \varphi_0^2 n_m^2 / 16\pi^2 T_0 C_p$ are the heating parameters, $Q_0 \sim \varphi_0^2 / \lambda^2$ where λ is the London penetration length. The ratio $S_J / S_A \sim \lambda^2 n_m$.

It seems at first glance that the direct Joule term caused by vortex (antivortex) motion always prevails. Really, for a sharp shape magnetic induction front the vortex-antivortex annihilation term (overlapping) which is proportional to the vortex (antivortex) density production $w_A \sim v N_+ N_-$ is small while vortex velocity $v \sim \nabla b$ is large. Therefore the direct, Joule term which is proportional both to the sum of the vortex and antivortex densities and to the square of the velocity $w_J = N S_J v^2$ significantly exceeds the annihilation term. *However, in our case, when the slope of the magnetic induction profile is small the annihilation term becomes essentially important.* In this case the overlapping (annihilation)

term is larger due to deep mutual penetration of vortices and antivortices over the interface area (see Fig 2). The vortex velocity in this case is small and it decreases the Joule term which is of the order of v^2 . Substituting functions N_+, N_- from the Eqs.(20),(21),(11) into Eq.(27) one obtains for the $w_J \simeq N_0 (\nabla b_0)^2 \simeq I^{5/3}$ and $w_A \sim N_0^2 \nabla b_0 \simeq I^{4/3}$ allowing to neglect in our consideration the Joule term which is relatively small $w_J/w_A \simeq I^{1/3} \ll 1$.

Here the dimensionless velocity v has the form

$$v = \exp(-U/T) \left| \frac{\partial b}{\partial x} \right| \quad (28)$$

while $\kappa \rightarrow t_0 \kappa_T / c_p (\Delta L)^2$ (here $\kappa_d = \kappa_T / C_p$ is the diffusion constant), $r \rightarrow t_0 / t_R$ are the dimensionless effective diffusion and relaxation coefficients correspondingly.

A. Small fluctuations

Looking for a solution of the form

$$b(x, y, t) = b_0(x) + \psi(x, y, t), \quad (29)$$

$$N(x, y, t) = N_0 + \zeta(x, y, t), \quad (30)$$

$$\Theta(x, y, t) = 1 + \theta(x, y, t), \quad (31)$$

where ψ, ζ and θ are the small perturbations of the form

$$\begin{pmatrix} \psi(x, y, t) \\ \zeta(x, y, t) \\ \theta(x, y, t) \end{pmatrix} = \begin{pmatrix} \psi(x) \\ \zeta(x) \\ \theta(x) \end{pmatrix} \exp(\lambda t + iky), \quad (32)$$

Here ψ_0, ζ_0 and θ_0 are constant amplitudes, λ is the rate grow of the perturbation and k is the wave vector in the y direction.

Taking into account the fluctuations of the vortex velocity

$$v_\alpha = -(1 + \theta) \gamma_{\alpha\beta} \frac{\partial b}{\partial x_\beta} \quad (33)$$

where θ/T_0 is the change of velocity due to thermal fluctuations [23] and $\gamma = \varphi_0 / 4\pi\eta$ (see Eq. 6)) one obtains

$$\begin{aligned} v_x &= v_x^0 + \delta v_x = -\gamma_{xx} \frac{\partial b_0}{\partial x} - \gamma_{xx} \frac{\partial \psi}{\partial x} - \gamma_{xx} \frac{\partial b_0}{\partial x} \theta - \gamma_{xy} \frac{\partial \psi}{\partial y} \\ v_y &= v_y^0 + \delta v_y = -\gamma_{yx} \frac{\partial b_0}{\partial x} - \gamma_{yx} \frac{\partial \psi}{\partial x} - \gamma_{yx} \frac{\partial b_0}{\partial x} \theta - \gamma_{yy} \frac{\partial \psi}{\partial y} \end{aligned} \quad (34)$$

Here $\delta v_{x,y}$ are the deviations from the steady state vortex velocity.

While the vortex velocity in the x direction is higher then in the y direction, we assume that $|v| \approx v_x$.

We neglect the influence of temperature fluctuations on heat capacity C_p and relaxation coefficient r because their calculations do not result in essential effects. We also neglect in the main order the change of the average temperature in the flux front area.

Substituting the perturbations in the form 32 into initial set of Eqs. (24)-(26), and use the stationary solution in the form Eqs. (14)-(15) (see Appendix I) one obtains from the Eq.(58) for the rate grow

$$\lambda^3 + \lambda^2 (\Gamma_1 + \Pi_1 k^2) + \lambda (\Gamma_2 + \Pi_2 k^2 + \Pi_3 k^4) + (\Pi_4 k^2 + \Pi_5 k^4) = 0, \quad (35)$$

where $\gamma_{xy} = \gamma_{yx}$ and

$$\begin{aligned} \Gamma_1 &= 2\gamma_{xx}I + r - \gamma_{xx}I^{\frac{4}{3}}S_A \\ \Pi_1 &= -\gamma_{yy}I^{\frac{1}{3}} + \kappa \\ \Gamma_2 &= 2\gamma_{xx}Ir \\ \Pi_2 &= 2(\gamma_{xy}^2 - \gamma_{xx}\gamma_{yy})I^{\frac{4}{3}} + 2\gamma_{xx}I\kappa - \gamma_{yy}I^{\frac{1}{3}}r + I^{\frac{5}{3}}S_A(\gamma_{xx}\gamma_{yy} - \gamma_{xy}^2) \\ \Pi_3 &= -\gamma_{yy}I^{\frac{1}{3}}\kappa \\ \Pi_4 &= 2(\gamma_{xy}^2 - \gamma_{xx}\gamma_{yy})rI^{\frac{4}{3}} + \gamma_{xx}\gamma_{xy}^2S_AI^{\frac{8}{3}} \\ \Pi_5 &= 2(\gamma_{xy}^2 - \gamma_{xx}\gamma_{yy})\kappa I^{\frac{4}{3}} \end{aligned} \quad (36)$$

The roots of these equations are presented in Appendix where the solution $\lambda_{1,2}$ are relevant.

(We consider only the nonuniform instability, hence solutions with $\text{Re } \lambda > 0$ at $k = 0$ are omitted). The onset of the nonuniform along the front instability is determined either by the conditions ($\text{Re } \lambda_1 = 0$)

$$(\text{Re } B_1)^2 = 4(\text{Re } C_1)(\text{Re } A_1) \quad (37)$$

$$\text{Re } C_1 < 0, (\text{Re } A_1) < 0 \quad (38)$$

giving the contact point at $\text{Re } \lambda_1 - k^2$ plane (see figs 3-5)

$$k^2 = -\frac{\text{Re } B_1}{2 \text{Re } A_1} \quad (39)$$

or at $C_1 = 0$ ($\text{Re } B_1 > 0, \text{Re } A_1 < 0$) resulting in the Mullens-Sekerka instability.

V. RESULTS

A. Strong Heating.

We start with a model case when the heating coefficient $S \rightarrow \infty$. In this case the parameters of the dispersion equation 36 have the form

$$\begin{aligned}\Gamma_1 &= -\gamma_{xx} I^{\frac{4}{3}} S_A; \Pi_1 = -\gamma_{yy} I^{\frac{1}{3}} + \kappa; \Gamma_2 = 2\gamma_{xx} I r; \Pi_2 = I^{\frac{5}{3}} S_A (\gamma_{xx} \gamma_{yy} - \gamma_{xy}^2); \\ \Pi_3 &= -\gamma_{yy} I^{\frac{1}{3}} \kappa; \Pi_4 = \gamma_{xx} \gamma_{xy}^2 S_A I^{\frac{8}{3}}; \Pi_5 = 2(\gamma_{xy}^2 - \gamma_{xx} \gamma_{yy}) \kappa I^{\frac{4}{3}}; \sqrt{\Gamma_1^2 - 4\Gamma_2} = \gamma_{xx} I^{4/3} S_A\end{aligned}\quad (40)$$

while the equation for $\text{Re } \lambda_2$ function reads (see Appendix II)

$$\text{Re } \lambda_2 = \frac{\gamma_{xy}^2 S_A I^{\frac{5}{3}}}{4r} k^2 - \frac{\gamma_{xy}^4 S_A^3 I^{\frac{11}{3}}}{64r^3} k^4 \quad (41)$$

The rapidly growing mode in this case $d^2 \text{Re } \lambda_2 / dk^2 = 0$ (maximum velocity of the mostly unstable mode) has the wave vector $k = 2\sqrt{2}r / \sqrt{3}\gamma_{xy} S_A I$ and the period of the pattern along the front (see fig.3)

$$d_y = \sqrt{\frac{3}{2}} \frac{\pi \gamma_{xy} S_A I}{r} \quad (42)$$

In the case of moderate and small heating parameter S_A , the results are strongly depends both on anisotropy and on the relation between other parameters and can be done numerically.

B. Moderate and weak heating.

In this case the equation for real part of the increment of the instability is determined by the equation (see Appendix II)

$$\text{Re } \lambda_1 = \text{Re } C_1 + k^2 \text{Re } B_1 + k^4 \text{Re } A_1; \quad (43)$$

It has been solved for different anisotropy, in-plane diffusion coefficients κ and relaxation coefficients r . The results are presented in figs. 4-6. In all of the curves at these figures the $\text{Re } C_1 < 0$, $\text{Re } B_1 > 0$ and $\text{Re } A_1 < 0$.

The instabilities in all of these cases has the form of contact one rather than the Mullens-Sekerka type. The nonuniform structure along the front appears with the period $d_y = 2\pi/k_c$ where k_c is the contact point of the $\text{Re } \lambda_1$ with k axis.

In fig.4 the $\text{Re } \lambda_1$ as a function of k^2 is shown for anisotropic superconductors with various in-plane diffusion constant κ . There is a critical heating parameter S and critical diffusion constant when the instability arises (curve 1), while the system becomes stable as the diffusion constants increase (curve 2,3). The relaxation constant r (the coefficient of the ballistic heat conductivity) also strongly affect the instability condition. In fig.5 the increment $\text{Re } \lambda$ versus k^2 for anisotropic superconductor with different relaxation constant r demonstrate that the instability appearing at relatively small constant r (see curve 1) disappears as the relaxation parameter grow (curves 2,3).

Fig.6 demonstrates that the anisotropy essentially affects the onset of the instability. The increment of instability $\text{Re } \lambda$ versus k^2 for different anisotropy. Curve 1 for isotropic superconductor ($\alpha = 0.9, \vartheta = \pi/4, \gamma_{xx} = \gamma_{yy} = 0.545, \gamma_{xy} = 0.055$) shows the instability at heating coefficient $S_A = 0.89$ while curve 2 demonstrates the lack of instability at heating coefficient $S_A = 0.8$. The curve 3 exhibits instability for anisotropic superconductor ($\alpha = 0.1, \vartheta = \pi/4, \gamma_{xx} = \gamma_{yy} = 0.55, \gamma_{xy} = 0.45$) and even more small heating coefficient $S_A = 0.1$.

VI. CONCLUSIONS

The standing flux-antiflux interface demonstrates instability due to the heat released by the flux-antiflux annihilation. In fact this is a well known Kelvin-Helmholtz (KH) instability appearing when different layers of liquid move with the opposite directed velocities [25]. In our case, however, vortex and antivortex "liquids" are moving as it is shown in Fig.7. The heat released by the annihilation enhances the vortex/antivortex velocities resulting in turbulence instability at the flux-antiflux interface. The rate grow dependence on the wave vector directed along the front showing the instability is presented in Figs.3-6 for different heating parameters and anisotropy of the superconducting materials. The characteristic size of the unstable pattern is determined either by the rapidly growing mode of the real part of the rate grow $d\lambda_2/dk$ (for strong heating parameter) (see fig.3) or at the contact mode for moderate and small heating (figs.4-6).

The theory predicts stability of the flux-antiflux front for any physical parameter of the

system (see Eqs. A12 from Appendix II). The physical reason for the instability is the result of growing temperature gradients along the front when vortices are moving with different velocities. The velocity of the flux flow vortices is very high and more rapid parts of the front can break it during the time of the instability. In particular for materials with typical parameters

$$B = 2000G, \eta = 5 \cdot 10^{-5} CGSE, v_F = 10^7 cm/sec, l = 10^{-8} cm, \xi = 10^{-6} cm \quad (44)$$

where B, η, v_F, l are the magnetic induction, viscosity, Fermi velocity and mean path length of the electron correspondingly, $n_m = B/\phi_0 \simeq 10^{10} cm^{-2}$ one obtains for characteristics units of time, space and diffusion constant κ_d (see Eq.10)

$$t_0 \simeq 5 \cdot 10^{-8} sec, \Delta L \simeq 10^{-4} cm, \kappa_d = \frac{(\Delta L)^2}{t_0} \kappa \simeq \kappa \quad (45)$$

the characteristic size of the interface in the dimension units $L_c \simeq \Delta L/I^{1/3}$.

For BCS superconductor where $\Delta \rightarrow 0$ the dimensional heating parameter $S_A \simeq n_m v_F^2 / T_0^2 \simeq \xi^2 n_m < 1$ (here $C_p \simeq m p_F T$, while ε_F and p_F are the Fermi energy and the momentum correspondingly. At low temperature $T < \Delta_0$, where $C_p \simeq (m p_F \Delta_0^{5/2} / T_0^{3/2}) \exp(-\Delta_0/T)$ the heating parameter S_A grows dramatically $S_A \simeq \frac{n_m v_F^2}{\Delta_0^2} \left(\frac{T_0}{\Delta_0}\right)^{1/2} \exp(\Delta_0/T) \gg 1$.

The heat parameter S_A is responsible for type of the instability. In particular at low temperatures ($T_0 < T_c$, where T_c is the critical temperature) the parameter S_A is large and the instability develops on Mullens-Sekerka scenario (see Fig.3) typical for dendritic instability [4]. On the other hand, at temperatures close to the critical, when the heating parameter $S_A < 1$, the instability emerges as a periodic pattern (see figs.4-6) [24].

Vortices and antivortices in the unstable pattern move with velocities (see Eq. (23)) $u \simeq 10^5 cm/sec$. If the difference of the "rapid" and "slow parts" of the front $\delta v \sim 10u$ the flux pattern might reach the microscopic magnitudes L for very short time of the instability development $\tau \simeq 30 \mu sec, L \approx 3 cm$. The heat fluctuation for this time cannot significantly relaxes because it moves along the front on distance $\delta y \simeq \sqrt{\kappa \tau} \sim 0.01 cm$.

The main results of this paper are presented in Figs 3-6 where the increments of the instability were drawn for various anisotropy parameter α , heat conductivities inside and across the sample (κ and r correspondingly) and heat annihilation coefficient S_A . We conclude that

heat released by the flux-antiflux annihilation results in instability of the interface separating fluxons and antifluxons areas even in the case of weak anisotropy of the superconductor.

On the other hand if the superconductor is strongly anisotropic, the instability emerges even for weak heat. In more experimentally common case of moderated heat and anisotropy, both these mechanisms work together creating the instability of the flux-antiflux front.

Our major conclusion is that the anisotropy of the superconducting layered structure alone cannot explain instability of the flux antiflux interface in weakly anisotropic materials as Nb and MgB_2 . From this point of view without the heating, the flux-antiflux front in the Nb superconductor ($\alpha \simeq 0.9$) without heating should be stable for any angle ϑ while strong heating destroys the front. If the heating caused by the vortex-antivortex annihilation is large then the vortex antivortex front instability should be detected in completely isotropic superconductors like MgB_2 (Fig.6, curve 1). In fact it should be noted that even small heating might be essentially important to cause the instability.

This theory is appropriate in the flux flow regime. The spatial disorder might affect the result by two different ways. It can both modify the linear profile of the magnetic induction at the front and affect the mechanism of the heat at the interface.

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VII. APPENDIX I

Substituting Eqs.(29)-(34) in the initial set of Eqs. (24)-(26), and use the stationary solution in the form Eqs. (14)-(15) one obtains for perturbations

$$\frac{\partial \psi}{\partial t} - N_0 \frac{\partial \delta v_x}{\partial x} - \zeta \frac{\partial v_x^0}{\partial x} - v_x^0 \frac{\partial \zeta}{\partial x} - N_0 \frac{\partial \delta v_y}{\partial y} - v_y^0 \frac{\partial \zeta}{\partial y} = 0 \quad (46)$$

$$\frac{\partial \zeta}{\partial t} - \frac{\partial b_0}{\partial x} \delta v_x - b_0 \frac{\partial \delta v_x}{\partial x} - v_x^0 \frac{\partial \psi}{\partial x} - b_0 \frac{\partial \delta v_y}{\partial y} - v_y^0 \frac{\partial \psi}{\partial y} + 2(N_0 \zeta - b_0 \psi) v_x^0 \quad (47)$$

$$\frac{\partial \theta}{\partial t} - \kappa \left(\frac{\partial^2 \theta}{\partial x^2} + \frac{\partial^2 \theta}{\partial y^2} \right) - 2S_A (N_0 \zeta - b_0 \psi) v_x^0 + r\theta = 0 \quad (48)$$

Looking for the solution of these equations in the form Eq.(32) one obtains near the interface where $b_0 = -I^{2/3}x$, $N_0 = I^{1/3}$ (see Eqs.(20)-(21)) the set of differential equations with uniform coefficients:

$$\left(\lambda - \gamma_{yy} I^{\frac{1}{3}} k^2 \right) \psi(x) + (\gamma_{xy} + \gamma_{yx}) ik I^{\frac{1}{3}} \frac{\partial \psi(x)}{\partial x} + \gamma_{xx} I^{\frac{1}{3}} \frac{\partial^2 \psi(x)}{\partial x^2} - \gamma_{yx} ik I^{\frac{2}{3}} \zeta(x) \quad (49)$$

$$- \gamma_{xx} I^{\frac{2}{3}} \frac{\partial \zeta(x)}{\partial x} - \gamma_{yx} ik I \theta(x) - \gamma_{xx} I \frac{\partial \theta(x)}{\partial x} = 0$$

$$- 2\gamma_{xy} ik I^{\frac{2}{3}} \psi(x) - 2\gamma_{xx} I^{\frac{2}{3}} \frac{\partial \psi(x)}{\partial x} + (\lambda + 2\gamma_{xx} I) \zeta(x) + \gamma_{xx} I^{\frac{4}{3}} \theta(x) = 0 \quad (50)$$

$$\gamma_{xy} ik I^{\frac{2}{3}} S_A \psi(x) + \gamma_{xx} I^{\frac{2}{3}} S_A \frac{\partial \psi(x)}{\partial x} - 2\gamma_{xx} I S_A \zeta(x) + \left(\lambda + r + \kappa k^2 - \gamma_{xx} I^{\frac{4}{3}} S_A \right) \theta(x) - \kappa \frac{\partial^2 \theta(x)}{\partial x^2} = 0 \quad (51)$$

These equations should be completed by the boundary conditions

$$\begin{pmatrix} \psi \\ \zeta \\ \theta \end{pmatrix}_{x=-x_c/2, x_c/2} = 0 \quad (52)$$

The functions $\psi(x)$, $\zeta(x)$ and $\theta(x)$ should be symmetrical and localized at the flux-antiflux interface where $x < x_c$ while $x_c \gg 1$ is the cutoff where these functions go to zero (see Fig.2 and Eq. (22)).

Looking for solution of Eqs. (46)-(51) in the form

$$\begin{pmatrix} \psi \\ \zeta \\ \theta \end{pmatrix} = \begin{pmatrix} (A_n \sin p_n x + B_n \cos p_n x) \\ (C_n \sin p_n x + D_n \cos p_n x) \\ (E_n \sin p_n x + F_n \cos p_n x) \end{pmatrix}; \quad (53)$$

one obtains equations for $A_n, B_n, C_n, D_n, E_n, F_n$ coefficients

$$\hat{\Lambda} \begin{pmatrix} A_n \\ B_n \\ C_n \\ D_n \\ E_n \\ F_n \end{pmatrix} = 0 \quad (54)$$

where matrix $\hat{\Lambda}$ reads

$$\begin{pmatrix} \lambda - \gamma_{yy} I^{\frac{1}{3}} k^2 - \gamma_{xx} I^{\frac{1}{3}} p_n^2 & -2\gamma_{xy} i p_n k I^{\frac{1}{3}} & -\gamma_{yx} i k I^{\frac{2}{3}} & p_n \gamma_{xx} I^{\frac{2}{3}} & -\gamma_{yx} i k I & p_n \gamma_{xx} I \\ p_n 2\gamma_{xy} i k I^{\frac{1}{3}} & \lambda - \gamma_{yy} I^{\frac{1}{3}} k^2 - \gamma_{xx} I^{\frac{1}{3}} p_n^2 & -\gamma_{xx} I^{\frac{2}{3}} p_n & -\gamma_{yx} i k I^{\frac{2}{3}} & -\gamma_{xx} I p_n & -\gamma_{yx} i k I \\ -2\gamma_{xy} i k I^{\frac{2}{3}} & 2p_n \gamma_{xx} I^{\frac{2}{3}} & \lambda + 2\gamma_{xx} I & 0 & \gamma_{xx} I^{\frac{4}{3}} & 0 \\ -2p_n \gamma_{xx} I^{\frac{2}{3}} & -2\gamma_{xy} i k I^{\frac{2}{3}} & 0 & \lambda + 2\gamma_{xx} I & 0 & \gamma_{xx} I^{\frac{4}{3}} \\ \gamma_{xy} i k I^{\frac{2}{3}} S_A & -\gamma_{xx} I^{\frac{2}{3}} S P_n & -2\gamma_{xx} I S_A & 0 & \lambda + r + \kappa k^2 - \gamma_{xx} I^{\frac{4}{3}} S_A + \kappa p_n^2 & \\ \gamma_{xx} I^{\frac{2}{3}} S_A P_n & \gamma_{xy} i k I^{\frac{2}{3}} S_A & 0 & -2\gamma_{xx} I S_A & 0 & \lambda + r + \kappa k^2 - \gamma_{xx} I^{\frac{4}{3}} S_A + \kappa p_n^2 \end{pmatrix} \quad (55)$$

where $p_n = (2n + 1) \pi / x_c$ is obtained from the boundary condition (52). The dangerous harmonic with $n = 0$ is responsible for instability. In this case $p_0 \sim x_c^{-1} \sim I^{1/3} \rightarrow 0$ for small slope of the magnetic induction at the interface ($I \ll 1$) this matrix can be simplified:

$$\begin{vmatrix} \lambda - \gamma_{yy} I^{\frac{1}{3}} k^2 & 0 & -\gamma_{yx} i k I^{\frac{2}{3}} & 0 & -\gamma_{yx} i k I & 0 \\ 0 & \lambda - \gamma_{yy} I^{\frac{1}{3}} k^2 & 0 & -\gamma_{yx} i k I^{\frac{2}{3}} & 0 & -\gamma_{yx} i k I \\ -2\gamma_{xy} i k I^{\frac{2}{3}} & 0 & \lambda + 2\gamma_{xx} I & 0 & \gamma_{xx} I^{\frac{4}{3}} & 0 \\ 0 & -2\gamma_{xy} i k I^{\frac{2}{3}} & 0 & \lambda + 2\gamma_{xx} I & 0 & \gamma_{xx} I^{\frac{4}{3}} \\ \gamma_{xy} i k I^{\frac{2}{3}} S_A & 0 & -2\gamma_{xx} I S_A & 0 & \lambda + r + \kappa k^2 - \gamma_{xx} I^{\frac{4}{3}} S_A & 0 \\ 0 & \gamma_{xy} i k I^{\frac{2}{3}} S_A & 0 & -2\gamma_{xx} I S_A & 0 & \lambda + r + \kappa k^2 - \gamma_{xx} I^{\frac{4}{3}} S_A \end{vmatrix} = 0 \quad (56)$$

and can be represented as generated Jordan matrix.

$$\begin{vmatrix} \hat{K} & 0 \\ 0 & \hat{K} \end{vmatrix} = 0 \quad (57)$$

where K is the 3×3 matrix

$$\hat{K} = \begin{vmatrix} \left(\lambda - \gamma_{yy} I^{\frac{1}{3}} k^2 \right) & \left(-\gamma_{yx} i k I^{\frac{2}{3}} \right) & \left(-\gamma_{yx} i k I \right) \\ \left(-2\gamma_{xy} i k I^{\frac{2}{3}} \right) & \left(\lambda + 2\gamma_{xx} I \right) & \left(\gamma_{xx} I^{\frac{4}{3}} \right) \\ \left(\gamma_{xy} i k I^{\frac{2}{3}} S_A \right) & \left(-2\gamma_{xx} I S_A \right) & \left(\lambda + r + \kappa k^2 - \gamma_{xx} I^{\frac{4}{3}} S_A \right) \end{vmatrix} \quad (58)$$

giving the equation for in the form

$$\lambda^3 + \lambda^2 (\Gamma_1 + \Pi_1 k^2) + \lambda (\Gamma_2 + \Pi_2 k^2 + \Pi_3 k^4) + (\Pi_4 k^2 + \Pi_5 k^4) = 0, \quad (59)$$

$$\Gamma_1 = 2\gamma_{xx} I + r - \gamma_{xx} I^{\frac{4}{3}} S_A$$

$$\Pi_1 = -\gamma_{yy} I^{\frac{1}{3}} + \kappa$$

$$\Gamma_2 = 2\gamma_{xx} I r$$

$$\Pi_2 = 2(\gamma_{xy}^2 - \gamma_{xx}\gamma_{yy}) I^{\frac{4}{3}} + 2\gamma_{xx} I \kappa - \gamma_{yy} I^{\frac{1}{3}} r + I^{\frac{5}{3}} S_A (\gamma_{xx}\gamma_{yy} - \gamma_{xy}^2)$$

$$\Pi_3 = -\gamma_{yy} I^{\frac{1}{3}} \kappa$$

$$\Pi_4 = 2(\gamma_{xy}^2 - \gamma_{xx}\gamma_{yy}) r I^{\frac{4}{3}} + \gamma_{xx} \gamma_{xy}^2 S_A I^{\frac{8}{3}}$$

$$\Pi_5 = 2(\gamma_{xy}^2 - \gamma_{xx}\gamma_{yy}) \kappa I^{\frac{4}{3}}$$

VIII. APPENDIX II

The solutions of the Eq. (59) at small $k \rightarrow 0$ read:

$$\begin{aligned} \lambda_0 &= -\frac{\Pi_4}{\Gamma_2} k^2 + A_0 k^4 \\ A_0 &= \frac{\Pi_2 \Pi_4}{\Gamma_2^2} - \frac{\Gamma_1 \Pi_4^2}{\Gamma_2^3} - \frac{\Pi_5}{\Gamma_2} \end{aligned} \quad (60)$$

$$\lambda_1 = C_1 + B_1 k^2 + A_1 k^4; \quad (61)$$

$$C_1 = \frac{1}{2} \left[-\Gamma_1 + \sqrt{\Gamma_1^2 - 4\Gamma_2} \right]; \quad (62)$$

$$B_1 = \frac{\left[\Gamma_1 - \sqrt{\Gamma_1^2 - 4\Gamma_2} \right] [\Gamma_1 \Pi_1 - \Pi_2] + 2 [\Pi_4 - \Gamma_2 \Pi_1]}{\Gamma_1 \left[\sqrt{\Gamma_1^2 - 4\Gamma_2} - \Gamma_1 \right] + 4\Gamma_2} \quad (63)$$

$$A_1 = \frac{B_1^2 \left[3\sqrt{\Gamma_1^2 - 4\Gamma_2} - \Gamma_1 \right] + \left[\sqrt{\Gamma_1^2 - 4\Gamma_2} - \Gamma_1 \right] [2B_1 \Pi_1 + \Pi_3] + 2 [\Pi_5 + B_1 \Pi_1]}{4\Gamma_2 + \Gamma_1 \left[\sqrt{\Gamma_1^2 - 4\Gamma_2} - \Gamma_1 \right]} \quad (64)$$

and

$$\lambda_2 = C_2 + B_2 k^2 + A_2 k^4; \quad (65)$$

$$C_2 = \frac{1}{2} \left[-\Gamma_1 - \sqrt{\Gamma_1^2 - 4\Gamma_2} \right];$$

$$B_2 = \frac{\left[\Gamma_1 + \sqrt{\Gamma_1^2 - 4\Gamma_2} \right] [\Pi_2 - \Gamma_1 \Pi_1] + 2 [\Gamma_2 \Pi_1 - \Pi_4]}{\Gamma_1 \left[\sqrt{\Gamma_1^2 - 4\Gamma_2} + \Gamma_1 \right] - 4\Gamma_2}; \quad (66)$$

$$A_2 = \frac{B_2^2 \left[3\sqrt{\Gamma_1^2 - 4\Gamma_2} + \Gamma_1 \right] + \left[\sqrt{\Gamma_1^2 - 4\Gamma_2} + \Gamma_1 \right] [2B_2 \Pi_1 + \Pi_3] - 2 [\Pi_5 + B_2 \Pi_1]}{\Gamma_1 \left[\sqrt{\Gamma_1^2 - 4\Gamma_2} + \Gamma_1 \right] - 4\Gamma_2} \quad (67)$$

Figure Captions

Fig.1 Geometry of the problem. ϑ is the angle between the x axis and the ab plain of the layered structure and c axis is perpendicular to the layers of the superconductor.

Fig.2 Structure of the vortex-antivortex interface.

Fig.3 Mullins-Sekerka instability for super large heat. The increment $\text{Re } \lambda$ versus k^2 (k is the wave vector along the flux-antiflux front).

Fig.4 The increment $\text{Re } \lambda$ versus k^2 for isotropic superconductors ($\alpha = 0.9, \vartheta = \pi/4$) with different in-plane diffusion constant κ . Curves 1,2,3 correspond to $\kappa = 0.1; 0.5; 1$ respectively. Here $\gamma_{xx} = \gamma_{yy} = 0.545, \gamma_{xy} = 0.055, I = 0.5, r = 0.0148, S_A = 0.89$. Instability disappears as the diffusion constant grows.

Fig.5 The increment $\text{Re } \lambda$ versus k^2 for anisotropic superconductor with different relaxation constant r . Curves 1,2,3 correspond to parameters $r = 0.049; 0.13; 2.66$ respectively. The instability disappears as the relaxation parameter grows. Here $\gamma_{xx} = \gamma_{yy} = 0.545, \gamma_{xy} = 0.055, \kappa = 0.1, I = 0.5, S_A = 0.89$.

Fig.6 The increment of instability $\text{Re } \lambda$ versus k^2 for different anisotropy. Curve 1 for isotropic superconductor ($\alpha = 0.9, \vartheta = \pi/4, \gamma_{xx} = \gamma_{yy} = 0.545, \gamma_{xy} = 0.055$) shows the instability at heating coefficient $S_A = 0.89$ while curve 2 demonstrates the lack of instability at heating coefficient $S_A = 0.8$. The curve 3 exhibits instability for anisotropic superconductor ($\alpha = 0.1, \vartheta = \pi/4, \gamma_{xx} = \gamma_{yy} = 0.55, \gamma_{xy} = 0.45$) and even more small heating coefficient $S_A = 0.1$. (Here $r = 0.0148, \kappa = 0.1, I = 0.5$).

Fig.7 Qualitative picture of the Kelvin-Helmholtz instability at the vortex-antivortex interface.

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